AD-A271 605

NAVAL UNDERSEA WARFARE CENTER
DETACHMENT NEW LONDON
NEW LONDON, CONNECTICUT 06320

TECHNICAL MEMORANDUM

AN ANALYTIC PERFORMANCE EVALUATION DESIGN
OF THE MFACP CW NORMALIZER

Date: 9 SEPT 1992

Prepared by:

Fyzodeen Khan
Code 3314 NUWC

Dr. Carl J. Wenk
A&T Engineering Technology
Center

DTIC
ELECTE
OCT 28 1993
S B D

93-25858

REPORT DOCUMENTATION PAGE

FORM APPROVED
OMB NO. 0704-0188

1. AGENCY USE ONLY (Leave blank)	2.. REPORT DATE 9 SEPT 1992	3. REPORT TYPE AND DATES COVERED TECHNICAL MEMORANDUM

4. TITLE AND SUBTITLE
ANALYTIC PERFORMANCE EVALUATION DESIGN OF THE MFACP CW NORMALIZER

5. FUNDING NUMBERS
C20020

6. AUTHOR(S)

FYZODEEN KHAN (CODE 3314) NUWC
DR. CARL J. WENK (A&T ENGINEERING TECHNOLOGY CENTER)

7. PERFORMING ORGANIZATION
NUWC, CODE 3314

8. PERFORMING ORGANIZATION REPORT NUMBER
TM 921205

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
HOWARD REICHEL, NAVSEA (PEO USW)

10. SPONSORING
NUWC

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
APPROVED FOR PUBLIC RELEASE.
DISTRIBUTION IS UNLIMITED.

12b. DISTRIBUTION CODE
SEE ATTACHED

13. ABSTRACT (Maximum 200 words)

AN ANALYTIC PERFORMANCE EVALUATION METHODOLOGY OF THE MID-FREQUENCY ACTIVE CLASSIFICATION PROCESSOR CONTINUOUS WAVE (MFACP CW) PHASE 1 NORMALIZER ALGORITHM IS PRESENTED. THIS NORMALIZER USES A RECURSIVE EXPONENTIAL FILTER STRUCTURE FOR ESTIMATION OF THE MEAN NOISE BACKGROUND LEVEL OF THE TEST CELL OF INTEREST. THE METHODOLOGY ALLOWS THE DETERMINATION OF THE EXPECTED VALUE AND VARIANCE OF THE NORMALIZER MEAN ESTIMATE WHICH ARE REQUIRED STATISTICS FOR THE DETERMINATION OF THE PROBABILITY OF FALSE ALARM P(F) AND DETECTION P(D) FROM WHICH THE RECEIVER OPERATING CHARACTERISTIC (ROC) CURVES ARE FORMED. THE SIGNAL PROCESSING PRIOR TO THE NORMALIZER CONSISTS OF DETERMINATION OF THE FAST FOURIER TRANSFORM (FFT) FROM 75% OVERLAPPED AND WINDOWED DATA (RECTANGULAR OR HANNING) FOLLOWED BY ENVELOPE DETECTION. THE ANALYTIC EXPRESSIONS DERIVED ARE FOR 75% OVERLAP BUT ARE APPLICABLE TO ANY OVERLAPPING AND WINDOWING PROCEDURE.

14. SUBJECT TERMS
MFACP I CW, RAYLEIGH, NORMALIZER, PROBABILITY OF DETECTION, PROBABILITY OF FALSE ALARM

15. NUMBER OF PAGES
21

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIIMITATION OF ABSTRACT

ABSTRACT

An analytic performance evaluation methodology of the Mid-Frequency Active Classification Processor Continuous Wave (MFACP CW) phase 1 normalizer algorithm is presented. This normalizer uses a recursive structure with an exponential filter for estimation of the mean background level of the cell under investigation. The methodology allows the determination of the expected value and variance of the normalizer mean estimate which are the essential parameters of the probability of false alarm P(F) and detection P(D) expressions from which the Receiver Operating Characteristic (ROC) curves can be computed. The signal processing prior to the normalizer consists of a Fast Fourier Transform (FFT) with 75% overlap windowed data (Rectangular or Hanning) and an envelope detector. The analytic expressions derived are done specifically for 75% overlap but are general enough so that different overlapping or windowing are applicable with minor modifications.

i

ADMINISTRATIVE INFORMATION

This memorandum was prepared under NUWC Project Number C20020, as part of the Normalizer Task of the Active Processing Initiative. The Task Leader is Fyzodeen Khan; the Principal Investigator is Fyzodeen Khan, Code 3314, the NUWC program manager is James Syck, 3112 and the program sponsor is Howard Reichel, NAVSEA (PEO USW).

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

1.0 - INTRODUCTION

This report provides a methodology for determining analytically the Receiver Operating Characteristic (ROC) curves of the Mid-Frequency Active Classification Processor (MFACP) Continuous Wave (CW) normalizer. The methodology is based on the determination of the statistics of the normalizer estimate under stationary and non-stationary noise conditions.

The model assumes the averaging time of the normalizer is long enough so that the probability density function of normalizer estimate μ converges to a Gaussian form as a consequence of the Central Limit Theorem. The dynamic noise background model described in section 4.0 has a sinusoidal variation in the Rayleigh parameter of the envelope detected narrowband Gaussian noise. This model of the shape of the background is quite general and in reality the shape of the background variation could be arbitrarily specified as required as long as the requirements given in 4.0 are met. Section 2.0 describes the false alarm and detection probabilities for each test cell under investigation which leads to the generation of the ROC curves. Section 3.0 develops the model for determining the mean and variance of the normalizer estimate and section 4.0 describes the background noise model.

2.0 - PERFORMANCE EVALUATION METHODOLOGY

The performance of each normalization algorithm is determined by evaluating the Receiver Operating Characteristic (ROC) curves. These curves quantify the performance of the algorithm as a function of Signal-to-Noise Ratio (SNR) and probability of detection, P(D), at different probabilities of false alarm, P(F).

A fundamental requirement of a normalization algorithm is a Constant False Alarm Rate output, CFAR. The MFACP maintains a constant detection threshold at every range point in the detection process. The CFAR requirement is achieved when the noise background is stationary, i.e., the statistics do not change with time. In non-stationary noise backgrounds with fixed detection thresholds the single bin P(F) and P(D) is dynamic and therefore the CFAR requirement is not met.

The test cell x is divided by the background mean level estimate μ generating the normalized output. Let λ denote the detection threshold, H_0 the hypothesis that the test cell x contains noise only and H_1 the hypothesis that test cell x contains a target echo (signal plus noise). (For notational convenience the time dependence for each successive test cell has been dropped). The following test is carried out to determine a detection:

$$\frac{x}{\mu} \underset{<}{\overset{\geq}{}} \lambda. \tag{2-1}$$

P(F) is the declaration that a true detection was made under hypothesis H_0, i.e., a detection is erroneously declared since x contained noise only. P(D) is the declaration that a true detection was made under hypothesis H_1, i.e., a detection is correctly declared since x contains signal plus noise. Under the noise only hypothesis H_0, the probability density function of the envelope of a narrowband Gaussian process has a Rayleigh form given in [7] as

$$f(x \mid H_0) = \frac{x}{\sigma^2} \exp\left(\frac{-x^2}{2\sigma^2}\right) u(x) \tag{2-2}$$

u(x) is the unit step function. For a narrowband signal in narrowband noise, i.e. signal plus noise hypothesis H_1, the probability density function of the test cell x is Rician and given by

$$f(x \mid H_1) = \frac{x}{\sigma^2} \exp\left(\frac{-(x^2 + A^2)}{2\sigma^2}\right) I_0\left(\frac{Ax}{\sigma^2}\right) u(x) \tag{2-3}$$

where $I_0(\cdot)$ is the modified Bessel function of order zero. The SNR in dB for the test cell x containing a target echo is

$$SNR = 10 \log_{10}\left(\frac{A^2}{2\sigma^2}\right).$$ (2-4)

It is assumed that the normalizer window size (averaging time) is large enough so that the convergence property of the Central Limit Theorem holds and that a Gaussian model for the normalizer estimate μ is valid. The probability density function of the normalizer estimate μ is [4-6]

$$f_\mu(\mu) = \frac{1/\sqrt{2\pi}\,\sigma_\mu}{F_G(m_\mu/\sigma_\mu)}\exp\left(\frac{-(\mu-m_\mu)^2}{2\sigma_\mu^2}\right)u(\mu)$$ (2-5)

where $F_G(\cdot)$ is the cumulative distribution of the standard normal zero mean, unit variance form and

$$m_\mu = E[\mu]$$
$$\sigma_\mu^2 = var(\mu).$$ (2-6)

P(F) and P(D) are also given in [4-6] as

$$P(F) = p\left(\frac{x}{\mu} \geq \lambda \mid H_0\right)$$
$$= \left[\frac{\exp\left(-\frac{1}{2}\frac{\lambda^2 m_\mu^2/\sigma^2}{1+\lambda^2\sigma_\mu^2/\sigma^2}\right)}{\sqrt{1+\lambda^2\sigma_\mu^2/\sigma^2}}\right]\left[\frac{F_G\left(\frac{m_\mu/\sigma_\mu}{\sqrt{1+\lambda^2\sigma_\mu^2/\sigma^2}}\right)}{F_G(m_\mu/\sigma_\mu)}\right]$$ (2-7)

and

$$P(D) = p\left(\frac{x}{\mu} \geq \lambda \mid H_1\right)$$

$$= \int_{-\infty}^{\infty} Q(A/\sigma, \mu\lambda/\sigma) f_\mu(\mu) d\mu \qquad (2\text{-}8)$$

$$= \frac{1}{\sqrt{\pi}} \sum_{n=1}^{Nh} w_n Q(A/\sigma, (\sigma_\mu \sqrt{2} z_n + m_\mu)\lambda/\sigma)$$

where $Q(q_0, q_1)$ is the Marqum-Q function for arguments q_0 and q_1 defined by

$$Q(q_0, q_1) = \int_{q_1}^{\infty} \psi \exp(-(\psi^2 + q_0^2)/2) I_0(q_0\psi) d\psi \qquad (2\text{-}9)$$

w_n and z_n are the Nh weights and zeros respectively associated with the Hermite polynomial expansion and are well tabulated in [9] for Nh up to 20. The polynomial expansion up to Nh = 6 provides very high accuracy. The Rayleigh parameter σ given in equations (2-7) and (2-8) is that of the test cell under investigation.

The analytic determination of the time history of the normalizer output mean and variance which are necessary in the determination of the ROC curves is given in section 3.0.

3.0 - MFACP/CW NORMALIZER FUNCTIONAL DESCRIPTION

Figure (3-1) is a block diagram of the signal processing prior to normalization. It consists of (a) a windowing function which is either a rectangle or Hanning, (b) a Fast Fourier Transform (FFT) with 75% overlap and (c) an envelope detector.

Figure (3-2) is a block diagram of the MFACP/CW normalizer. The mean level at each range bin for a given doppler channel is estimated by an exponential filter. The min function (minimum of two values) is used for large data outlier rejection so that the mean background level estimate is not biased. The given clipping threshold λ_c of 2.56 provides an optimal rate limitation [2, 3] of 0.76 dB per FFT update.

The mean background level estimate in each doppler channel at time k, denoted by $z(k)$, is the sum of the scaled input at time k, $\bar{x}(k)$, and the scaled mean background level output at time k-1, $z(k-1)$. The input data to the exponential filter, $\bar{x}(k)$, is the minimum of the envelope detected output at time k, $x(k)$, and the mean background level estimate $z(k-1)$. The exponential filter recursively updates the mean background level estimate at each time sample k using the clipped data at the output of the minimum function.

To further reduce the variance of the mean background level estimate, 5 doppler channels are averaged before generating the final normalizer estimate. The average of the five doppler channels, $z'(k)$, is symmetric about the channel of interest; i.e. 2 adjacent channels on either side of the channel of interest are used. To nullify the bias in the mean background estimate $z'(k)$ when the test cell of interest contains a target, a range gap symmetric about and centered on the test cell is normally used. The MFACP/CW normalizer is one-sided (asymmetric) and implements the gap by normalizing the test cell with the mean background estimate delayed by 4 samples, $z'(k-4)$.

In the analysis that follows the mean and variance of the output, $z'(k)$, are generated. The dynamic behavior of these two values determine the ROC curves at each range bin in nonstationary noise environments.
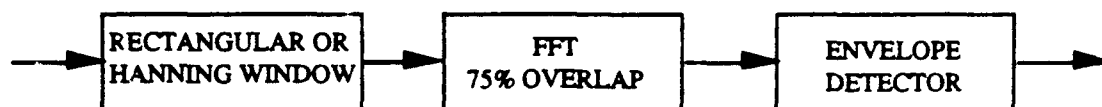
```
┌─────────────────┐     ┌─────────────┐     ┌─────────────┐
│  RECTANGULAR OR │     │     FFT     │     │  ENVELOPE   │
│  HANNING WINDOW │ ──▶ │ 75% OVERLAP │ ──▶ │  DETECTOR   │ ──▶
└─────────────────┘     └─────────────┘     └─────────────┘
```

Figure (3-1) - Signal processing prior to normalization.

3.0.1 - MFACP/CW CORRELATION COEFFICIENTS

For overlapped FFT processing as shown in figure (3-1) above the correlation coefficients between successive updates [1] is

$$\rho(\Omega) = \frac{\sum_{n=0}^{\Omega N-1} W(n)\, W(n+(1-\Omega)N)}{\sum_{n=0}^{N-1} W^2(n)}. \tag{3-1}$$

Ω is the fractional overlap and $W(n)$ is the window function. Figure (3-3) shows the sequencing of the data through the FFT for 75% overlap. After 4 updates the new input sequence to the FFT has no data points in common with the original input sequence.

For the four updates the fractional overlap Ω is 0.75, 0.5, 0.25 and 0 respectively for i = 1, 2, 3, 4. The correlation coefficient at the envelope detector output is given in [5, 6] as

$$r(i) = \frac{\rho^2(i)/4}{4/\pi - 1}\left\{1 + \frac{\rho^2(i)}{4}\sum_{n=0}^{\infty}\left[\frac{\prod_{k=1}^{n+1}(2k-1)}{2^n(n+1)!}\right]^2 \rho^{2n}(i)\right\} \tag{3-2}$$

where i = 1, 2, 3, 4 corresponds to the FFT update. The infinite sum in equation (3-2) may be truncated to a small finite number, Nn, of terms producing great accuracy (Nn ≤ 30). Table (3-1) lists the correlation coefficients at the outputs of the FFT and envelope detector for both the rectangular and Hanning windows. Figure (3-4) is a plot of the correlation coefficients comparing both window types at the FFT and envelope detector outputs.
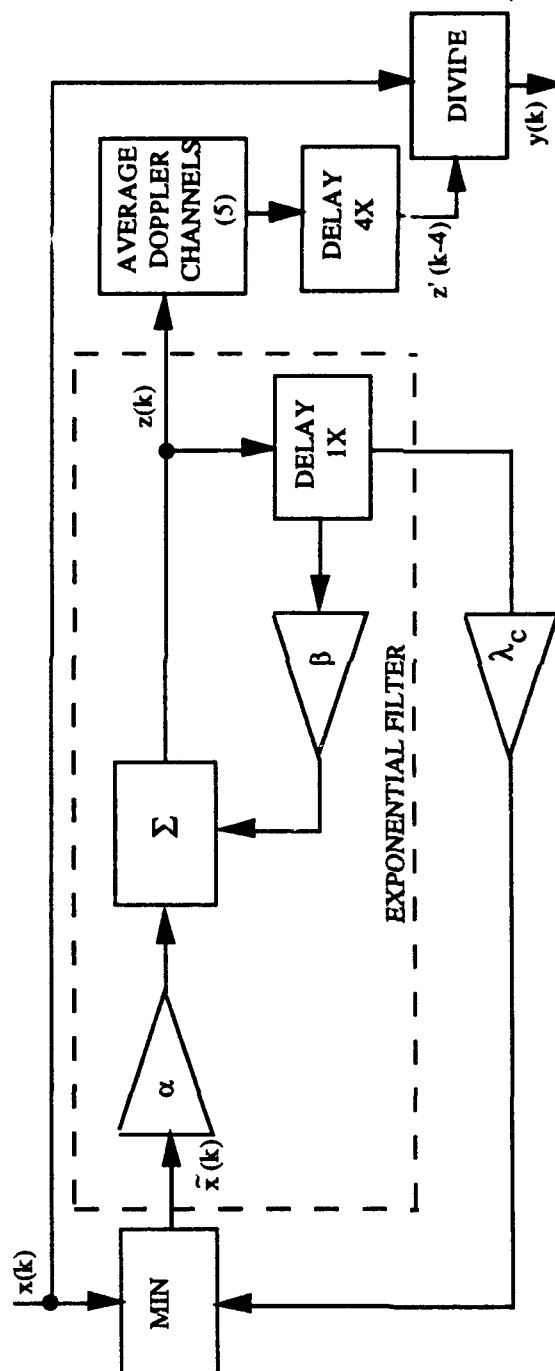
Figure (3-2) - Block diagram of MFACP/CW normalizer showing the minimum function, exponential filter, average over doppler cells and 4 sample delay.

update, i	ρ_R, rectangle	r_R, rectanlge	ρ_H Hanning	r_H Hanning
0	1.0	1.0	1.0	1.0
1	0.75	0.5361	0.6592	0.4098
2	0.50	0.2326	0.1667	0.0255
3	0.25	0.0574	0.0075	0.0001
4	0	0	0	0

Table (3-1) - Correlation coefficients at output of FFT and envelope detector.

The noise background at the FFT output is assumed to be a zero mean Gaussian process, N(0, σ^2). At the output of the envelope detector the noise is a Rayleigh process [7] with Rayleigh parameter σ^2 equal to the variance of the Gaussian noise. The density function for a Rayleigh random variable is

$$p(x) = \frac{x}{\sigma^2} \exp\left(\frac{-x^2}{2\sigma^2}\right) u(x) \tag{3-3}$$

where u(x) is the unit step function. Figure (3-5) is an example plot of equation (3-3) for the Rayleigh parameter $\sigma^2 = 1$. The mean and variance of a Rayleigh random variable x are

$$\mu_x = \sqrt{\pi/2}\ \sigma$$
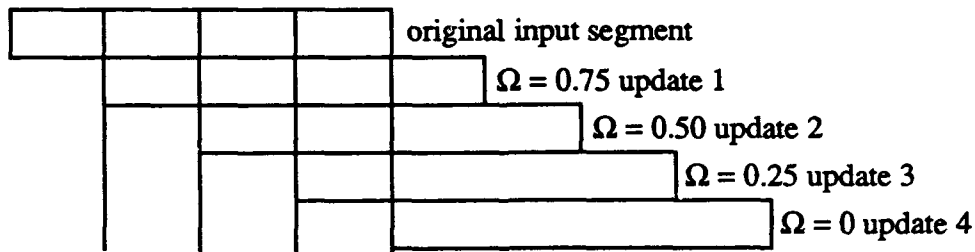$$\sigma_x^2 = (2 - \pi/2)\sigma^2.$$

Figure (3-3) - Sequencing of data segments for 75% overlap FFT processing.

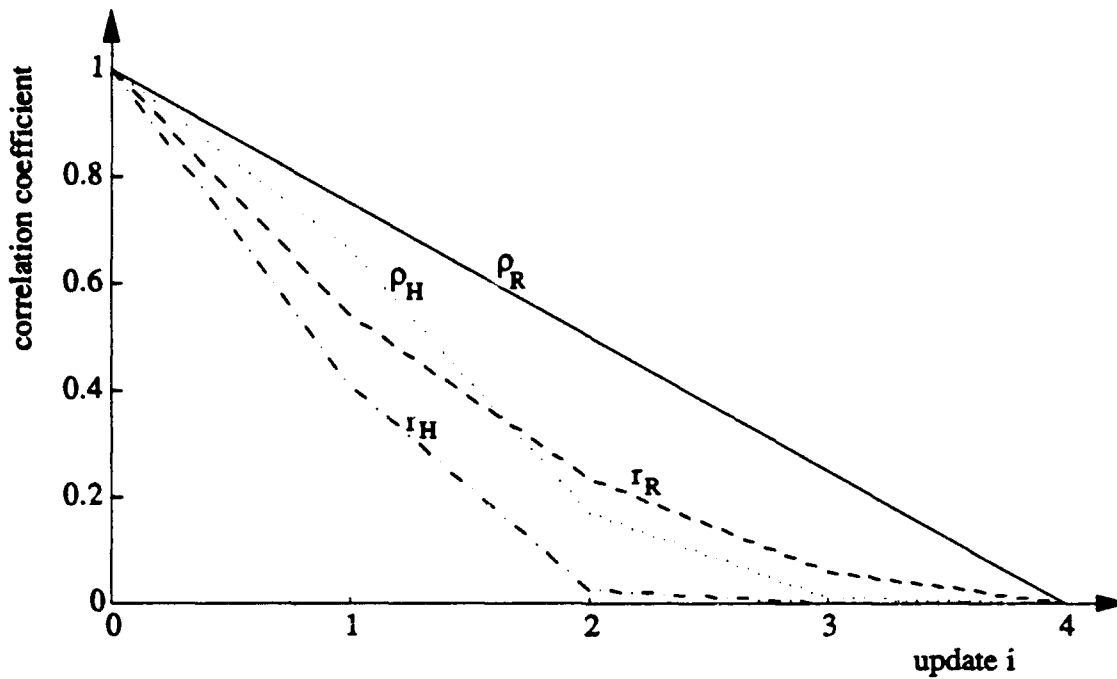Figure (3-4) - Plot of the correlation coefficients of FFT update at the output of the overlapped FFT (equation A1) and envelope detector (equation 3-2) for a rectangular and Hanning window as given in table (3-1).
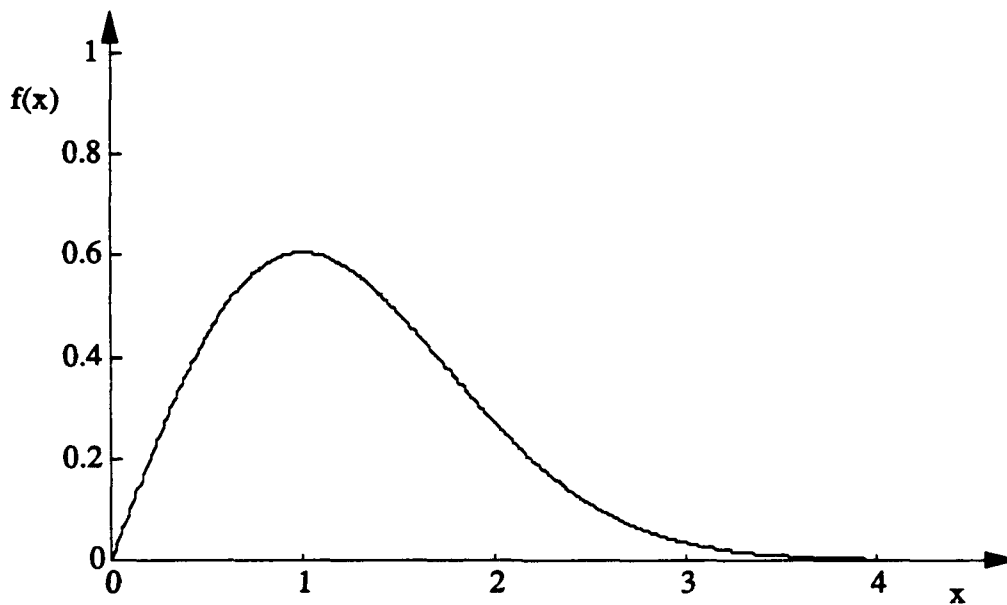

Figure (3-5) - Example plot of the Rayleigh density function of the samples at the output of the envelope detector which are input to minimum function for Rayleigh parameter $\sigma^2 = 1$.

3.0.2 - EXPONENTIAL FILTER

The exponential filter [2, 3] output at time k (see figure (3-2)) denoted by z(k) is

$$z(k) = \alpha \tilde{x}(k) + \beta z(k-1) \tag{3-4}$$

where $\alpha = \dfrac{1}{N+1}, \beta = \dfrac{N}{N+1}$ and N = 15.

Modifying z(k) to incorporate M+1 previous input values gives

$$z(k) = \sum_{i=0}^{M} \alpha \beta^i \tilde{x}(k-i) + \beta^{M+1} z(k-M-1). \tag{3-5}$$

The mean of the exponential filter output z(k) denoted by $\mu_z(k) = E[z(k)]$ is

$$\begin{aligned} \mu_z(k) &= E[z(k)] = \alpha E[\tilde{x}(k)] + \beta E[z(k-1)] \\ &= \alpha \mu_{\tilde{x}}(k) + \beta \mu_z(k-1) \\ &= \alpha \sum_{i=0}^{M} \beta^i E[\tilde{x}(k-i)] + \beta^{M+1} E[z(k-M-1)]. \end{aligned} \tag{3-6}$$

The variance at the exponential filter output denoted by $\sigma_z^2(k)$ is given by

$$\begin{aligned} \sigma_z^2(k) &= E[z^2(k)] - E[z(k)]^2 \\ &= \alpha^2 \sigma_{\tilde{x}}^2(k) + \beta^2 \sigma_z^2(k-1) + 2\alpha\beta \, \mathrm{cov}(\tilde{x}(k), z(k-1)). \end{aligned} \tag{3-7}$$

The covariance between the present input x(k) and the previous output z(k-1), denoted by cov(x(k), z(k-1)) in equation (3-7), is by definition [8]

$$\mathrm{cov}(\tilde{x}(k), z(k-1)) = E[\tilde{x}(k) \, z(k-1)] - E[\tilde{x}(k)] \, E(z(k-1)).$$

Substituting the relationship given in equation (3-5) for the previous M+1 input values the covariance expression becomes

$$cov(\tilde{x}(k), z(k-1)) = \alpha\sum_{i=0}^{M}\beta^i\left(E[\tilde{x}(k)\tilde{x}(k-1-i)] - E[\tilde{x}(k)]\,E[\tilde{x}(k-1-i)]\right)$$

$$= \alpha\sum_{i=0}^{M}\beta^i\,r(i)\,\sigma_{\tilde{x}}(k)\,\sigma_{\tilde{x}}(k-1-i) \tag{3-8}$$

where also by definition [8]

$$cov(\tilde{x}(k), \tilde{x}(k-1-i)) = r(i)\,\sigma_{\tilde{x}}(k)\,\sigma_{\tilde{x}}(k-1-i)$$

was used. Substituting equation (3-8) back into (3-7) and the fact that the correlation coefficient $r(i)$ goes to zero after $M+1 = 4$ updates the exponential filter output variance becomes

$$\sigma_z^2(k) = \alpha^2\sigma_{\tilde{x}}^2(k) + \beta^2\sigma_z^2(k-1) + 2\alpha^2\sum_{i=1}^{M+1}\beta^i\sigma_{\tilde{x}}(k)\sigma_{\tilde{x}}(k-i)r(i). \tag{3-9}$$

Equations (3-6) and (3-9) are general expressions _ ···ing the recursive relationship for the mean and variance of the exponential filter output in either stationary or non-stationary noise backgrounds. If the filter is operating in a stationary noise field, the filter goes to steady state and the output mean and variance becomes

$$\mu_z(k) = \mu_{\tilde{x}}(k) = \mu_{\tilde{x}}$$

$$\sigma_z^2(k) = \frac{\alpha^2\sigma_{\tilde{x}}^2}{1-\beta^2}\left(1 + 2\sum_{i=1}^{M+1}\beta^i r(i)\right) = \sigma_z^2. \tag{3-10}$$

To reduce the variance of the exponential filter output 5 doppler channels are averaged. The variance is thus reduced to, by assuming the 5 doppler channels are independent and identically distributed,

$$\sigma_{z'}^2(k) = \sigma_z^2(k)/5. \tag{3-11}$$

The input test cell under investigation is divided by the mean estimate delayed by 4 samples producing the normalized output

$$y(k) = \frac{x(k)}{z'(k-4)}. \tag{3-12}$$

3.0.3 - MFACP/CW MINIMUM FUNCTION

The minimum function chooses the smaller of the Rayleigh output from the envelope detector at time k and the scaled output of the exponential filter at time k-1. This function essentially shears (clips) the data to eliminate large data outliers from corrupting the mean level estimate and provides a rate limitation of 0.76 dB per FFT update [2, 3]. The density function of a clipped Rayleigh random variable is

$$p_{\tilde{x}}(\tilde{x}) = \frac{\tilde{x}}{\sigma^2} \exp\left(\frac{-\tilde{x}^2}{2\sigma^2}\right) [u(\tilde{x}) - u(\tilde{x} - \lambda_c)] + \exp\left(\frac{-\lambda_c^2}{2\sigma^2}\right) \delta(\tilde{x} - \lambda_c) \tag{3-13}$$

where $\delta(\tilde{x} - \lambda_c)$ is the unit impulse function located at $\tilde{x} = \lambda_c$. The amplitude of the delta function is the probability of the Rayleigh random variable exceeding the clipping threshold. The mean and variance of the clipped Rayleigh random variable are given by

$$\mu_{\tilde{x}} = E[\tilde{x}] = \left(\frac{\pi}{2}\right)^{1/2} \sigma \, \text{erf}\left(\frac{\lambda_c}{\sqrt{2\sigma^2}}\right)$$

$$E[\tilde{x}^2] = 2\sigma^2 \left(1 - \exp\left(\frac{-\lambda_c^2}{2\sigma^2}\right)\right) \tag{3-14}$$

$$\sigma_{\tilde{x}}^2 = E[\tilde{x}^2] - E[\tilde{x}]^2.$$

$\text{erf}(\phi)$ is the standard error function defined in [9] as

$$\text{erf}(\phi) = \frac{2}{\sqrt{\pi}} \int_0^\phi \exp(-\psi^2) \, d\psi. \tag{3-15}$$

A plot of the clipped Rayleigh density function specified by equation (3-13) is shown in figure (3-5) for $\sigma^2 = 1$ and $\lambda_c = 2$. The height of the delta function is equal to the probability of the Rayleigh random variable $x \geq \lambda_c$.

3.0.4 - MFACP/CW EXPONENTIAL FILTER OUTPUT NOISE MODEL

The exponential filter output mean and variance is determined by combining the results from 3.0.2 and 3.0.3. To determine the mean and variance of the exponential
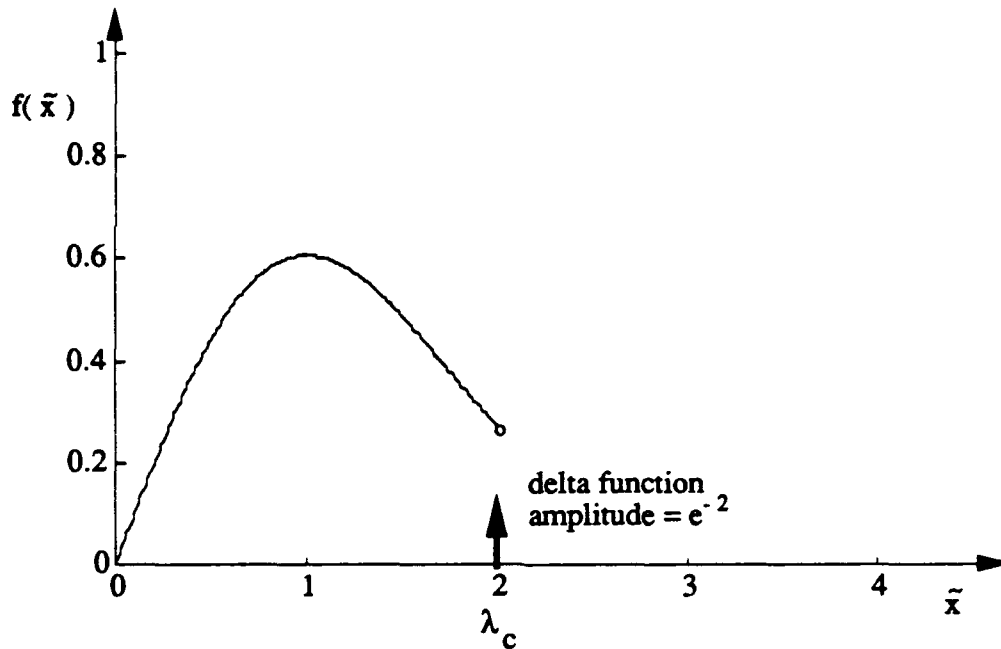
Figure (3-6) - Example plot of the density function of the clipped Rayleigh random variables at the output of the minimum function for Rayleigh parameter $\sigma^2 = 1$ and $\lambda_c = 2$. The amplitude of the delta function is equal to the probability of the Rayleigh random variable $x \geq \lambda_c$.

filter output the algorithm as shown in figure (3-2) is combined with equations (3-6, 3-9) and (3-14). This model results in the following expressions for the mean and variance of the exponential filter output at each time k:

$$\mu_z(k) = \alpha \min(\mu_x(k), 2.56\mu_z(k-1)) + \beta \mu_z(k-1)$$
$$= m_\mu \tag{3-16}$$

$$\sigma_{\tilde{x}}^2(k) = 2\sigma_x^2(k) \left\{ 1 - \exp\left(\frac{-(2.56\mu_z(k-1))^2}{2\sigma_x^2(k)} \right) \right\}$$
$$- \left(\frac{\pi}{2} \right)\sigma_x^2(k) \left(\text{erf}\left(\frac{2.56\mu_z(k-1)}{\sqrt{2}\,\sigma_x(k)} \right) \right)^2 \tag{3-17}$$

$$\sigma_z^2(k) = (\alpha\sigma_{\tilde{x}}(k))^2 + (\beta\sigma_z(k-1))^2 + 2\alpha^2 \sum_{i=1}^{M+1} \beta^i \sigma_{\tilde{x}}(k)\sigma_{\tilde{x}}(k-i)r(i)$$
$$= \sigma_\mu^2. \tag{3-18}$$

3.0.5 - EFFECTIVE AVERAGING TIME

The effective normalizer averaging time is determined as the number of independent samples that go into the computation of the mean estimate for random error reduction. For stationary noise with $\sigma = 1$ the mean and variance of the clipped Rayleigh random variable using equation (3-14) evaluates to

$$
\mu_{\bar{x}} = E[\bar{x}] = \sqrt{\pi/2}\,\text{erf}\left(\frac{2.56\sqrt{\pi/2}}{\sqrt{2}}\right) = 1.25163
$$

$$
\sigma_{\bar{x}}^2 = 2\left(1 - \exp\left(\frac{-2.56^2(\pi/2)}{2}\right)\right) - 1.25163^2 = 0.42178. \tag{3-19}
$$

Substituting now into equation (3-10) gives the variance of the estimate before doppler averaging as

$$
\sigma_z^2 = \frac{0.42178}{31}(1.81336) = \frac{0.42178}{17}. \tag{3-20}
$$

The effective averaging time of the recursive structure of the normalizer is determined by comparing to an equivalent window structure as would occur in a split window scheme. Looking at equation (3-11) the equivalent length of the split window block averager is given by

$$
N_{IID} = \frac{\sigma_{\bar{x}}^2}{\sigma_{z'}^2} = \frac{(1-\beta^2)N_{DOPPLER}}{\alpha^2(1 + 2\sum_{i=1}^{M+1}\beta^i r(i))} = \frac{(31)(5)}{1.81336} = 85 \tag{3-21}
$$

where $N_{DOPPLER}$ is the number of Doppler channels averaged. Therfeore by averaging the 5 doppler channels the effective averaging time is increased from $N_{IID} = 17$ to 85 samples.

4.0 - BACKGROUND NOISE MODEL

The noise nonstationarity considered here [5] allows for an arbitrary positive sample-to-sample variation in the Rayleigh parameter σ_k. The mean of the single noise cell varies from sample-to-sample as

$$\mu_{x_k} = E[x_k] = \sqrt{\pi/2}\,\sigma_k \qquad (4\text{-}1)$$

This type of nonstationarity corresponds to a sample-to-sample change in the total noise power without necessarily a variation in the power spectrum with frequency. The latter nonstationarity, involving a spectral shape change without a total power change, does not affect the first order statistic, $E[x_k]$, but does affect the sample-to-sample covariance

$$v_{ij} = \text{cov}(x_i, x_j) = r_{ij}\,\sigma_{x_i}\sigma_{x_j} \qquad (4\text{-}2)$$

where $\sigma_{x_k}^2$ is the variance of sample x_k where $k = i, j$

$$\sigma_{x_i}^2 = (2 - \pi/2)\,\sigma_i^2 \qquad (4\text{-}3)$$

by changing the correlation coefficient, r_{ij},

$$r_{ij} = \frac{v_{ij}}{\sigma_{x_i}\sigma_{x_j}} = \frac{E[x_i x_j] - E[x_i]E[x_j]}{\sigma_{x_i}\sigma_{x_j}} \qquad (4\text{-}4)$$

this total power nonstationarity also affects v_{ij} through σ_{x_i} and σ_{x_j}. Thus, noise nonstationarity is modeled by a total power nonstationarity affecting both m_{x_i} and v_{ij}, and a spectral shape nonstationarity with frequency affecting only v_{ij}. In stationary noise backgrounds, the correlation coefficients r_{ij} does not vary with time. In nonstationary noise, the Rayleigh parameter σ_i change from sample-to-sample and the correlation coefficients may change as well.

In shallow water environments it has been observed in real data that the [6] background variation is oscillatory in nature and can vary by as much as 20 dB in the space of a few seconds. In convergence zone regions, the background may vary by as much as 15 to 20 db per second and has a pedestal shape to the variation. The effects on normalizer performance is captured by considering a generalized sinusoidal background variation model where both the amplitude and period of the variation may be arbitrarily specified.

Let the variation in the Rayleigh parameter as a function of time be defined by

$$\sigma(t) = \sigma_0 - \sigma_{min} \sin(2\pi t / T) \tag{4-5}$$

where d is the total power change in dB and T is the period of the sinusoid. Also let d be defined by

$$d = 20 \log_{10}(\sigma_{max} / \sigma_{min}) \tag{4-6}$$

or

$$10^{d/20} = \sigma_{max} / \sigma_{min} \tag{4-7}$$

so that at the minimum and maximum points in the sinusoidal variation

$$\sigma_{max} = \sigma_0 - \sigma_1 \sin(3\pi / 2) = \sigma_0 + \sigma_1$$
$$\sigma_{min} = \sigma_0 - \sigma_1 \sin(\pi / 2) = \sigma_0 - \sigma_1. \tag{4-8}$$

Equating equations (4-7) and (4-8) gives

$$10^{d/20} = \frac{\sigma_0 + \sigma_1}{\sigma_0 - \sigma_1}$$
$$\sigma_1 = \left(\frac{10^{d/20} - 1}{10^{d/20} + 1}\right)\sigma_0 \tag{4-9}$$

and finally the variation of the Rayleigh parameter for the sinusoid can be written as

$$\sigma(t) = \sigma_0\left[1 - \left(\frac{10^{d/20} - 1}{10^{d/20} + 1}\right)\sin\left(\frac{2\pi t}{T}\right)\right]. \tag{4-10}$$

Appendix R - References

1. Harris, F. J. - " On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform," Proceedings Of The IEEE, vol. 66, No. 1, January 1978.

2. Gilbert, D. C. - "Algorithm and Software Specification for ACP CW Detection and Tracking," Analysis and Tecnology, Inc., Report no. P-4569-1-91, 31 Dec 91.

3. Gilbert, D. C. - "Automatic Detection Algorithm for ACP CW Processing," Analysis and Tecnology, Inc., Tech. Memo. 90:32/27-34A, 30 Nov 90.

4. Wenk, C. J. - "Analytical Detection and False Alarm Probabilities for Narrowband SONAR Normalizers in Nonstationary Noise Backgrounds (PRELIMINARY)," Analysis and Technology, Inc., Tech Memo. 91:32/27-34A, 20 Nov 91.

5. Wenk, C. J. - "Analytical Window Statistics of One and Two-Pass Mean Value Narrowband Normalizers in Nonstationary Noise Backgrounds (PRELIMINARY)," Analysis and Technology, Inc., Tech Memo. 91:32/27-32A, 13 Nov 91.

6. Wenk, C. J. - "An Analytical Design and Evaluation Methodology for Active SONAR Normalizers (PRELIMINARY),"Analysis and Tecnology, Engineering Technology Center, Report no. P-4578-5-92, 15 March 92.

7. Whalen, A. - Detection of Signals in Noise, Academic Press, Inc., New York 1971.

8. Papoulis, A. - Probability, Random Variables and Stochastic Processes, McGraw-Hill, Inc., second edition.

9. Abramowitz, M., Stegun, I. ed. - Handbook of Mathematical Functions, U. S. Department of Commerce, National Bureau of Standards, Applied Mathematics Series, No. 55, Washington, D. C., June 1964.